

Institute for Quantum Information

Findings – 2007-08

Quantum information science is an exciting emerging field that addresses how fundamental physical laws can be harnessed to dramatically improve the acquisition, transmission, and processing of information. The primary goal of the Institute for Quantum Information (IQI) is to carry out and facilitate research in quantum information science. Our research covers six broad areas: (1) Quantum algorithms that achieve speedups relative to classical algorithms, and limits on such algorithms. (2) Quantum cryptographic protocols, and other types of communication using quantum states. (3) Quantum entanglement and the theory of transformations among quantum states. (4) Protection of quantum information using quantum error correcting codes, fault tolerant protocols for quantum information processing, and control of quantum systems. (5) Theory and practice regarding physical implementations of quantum information processing. (6) Connections between quantum information science and other aspects of fundamental physics.

Since our last annual report in May 2007, IQI participants have produced 41 publications, which we summarize here.

Quantum algorithms and quantum complexity

Span-program-based quantum algorithm for evaluating formulas. Reichardt, with Spalek, gave a quantum algorithm for evaluating “span programs” (a span program is a certain linear-algebraic way of specifying a classical function) [1]. In particular, the algorithm optimally evaluates balanced formulas over an extended gate set including all two-bit and three-bit gates (such as NAND and 3-majority). The main new tool in their analysis is a correspondence between span programs and weighted bipartite graphs — a span program’s evaluation corresponds to an eigenvalue-zero eigenvector of the associated graph. A quantum computer evaluates the span program by applying spectral estimation to the graph.

Wavefunction preparation using a quantum computer. Kitaev and Webb described a set of algorithms for preparing certain quantum states (representing continuous functions) on qubit ensembles [2]. In particular, they presented an algorithm that prepares an ensemble of qubits into a wavefunction corresponding to a multidimensional Gaussian. This algorithm uses a very simple subroutine to prepare a set of independent one-dimensional Gaussian states and

then employs a reversible transformation to produce an arbitrary Gaussian. These Gaussian states have applications in multidimensional resampling, where wavepacket stretching and squeezing is simulated using a fixed grid. This research is part of a broader effort to develop techniques for digital simulation on quantum computers.

Optimal quantum adversary lower bounds for ordered search. Childs, with Lee, found the exact value of the best possible quantum adversary lower bound for a symmetrized version of ordered search [3]. The goal of the ordered search problem is to find a particular item in an ordered list of n items, and the query complexity of the symmetrized version differs from that of the unsymmetrized version by at most 1. They showed that the best lower bound for ordered search that can be proved by the adversary method (even if negative weights are allowed) is $\frac{1}{\pi} \ln n + O(1)$.

The complexity of the Local Consistency problem. Local Consistency is the qubit version of the N -representability problem in quantum chemistry. Liu found a novel reduction from Local Consistency to Local Hamiltonian, using strong duality of semidefinite programs [4, 5]. This reduction differs from previous work in that it preserves the structure of the underlying physical system. This allows one to study special classes of physical systems, such as 1-D and “stoquastic” systems, which are not necessarily QMA-hard; Liu found that Local Consistency and Local Hamiltonian still have equivalent complexity in these special cases [5].

Hardness of a single-shot energy measurement of a product state in a translation-invariant spin chain. Wocjan and Zhang, with Janzing, showed that the output of an arbitrary quantum circuit can be determined by performing a single-shot energy measurement on a computational basis state, where the energy is determined by the Hamiltonian of a one-dimensional qudit chain [6]. Thus such energy measurements are as hard as the realization of any quantum computation. Here a “measurement” is a procedure that samples from the spectral measure induced by the observable and the state under consideration (the post-measurement state is irrelevant), and the required measurement accuracy scales inverse polynomially with the size of the simulated quantum circuit.

Lower bound methods for quantum one-way communication complexity. Communication complexity is a powerful tool for deriving lower bounds in numerous areas of classical theoretical computer science, which raises the fundamental question: how large is the gap between the classical and quantum communication complexity for evaluation of total functions? The answer remains elusive, despite much effort. In [7], Zhang showed that all the known quantum lower bound methods for the one-way communication model can be exponentially weak, explaining past failures to settle the question, and highlighting the need for new lower bound methods. Then, with Jain, Zhang derived new lower bounds for quantum one-way distributional communication complexity in terms of a widely-used measure, the rectangle bound [8]. They also derived new *upper* bounds for classical one-way communication complexity in terms of mutual information in a hard distribution, improving on the best known previous results for total functions.

Quantum communication and quantum cryptography

Optimal quantum source coding with quantum information at the encoder and decoder. Yard, with Devetak, found the necessary and sufficient entropic conditions that allow Alice to transfer quantum information to Bob with negligible loss of fidelity [9]. Their conditions quantify the value of quantum entanglement for redistributing quantum states, and also show that Alice can exploit side information to improve the information transfer rate. Specifically, they consider an initial pure state of four systems $ACBR$, where Alice holds AC , Bob holds B , and R represents an environment correlated with ACB , and they prove that Alice can redistribute C to Bob with negligible error (asymptotically) if and only if $Q < I(R; C|B)/2$ and $Q + E < H(C|B)R$. Here Q is the number of qubits sent from Alice to Bob, E is the number of ebits of entanglement initially shared by Alice and Bob that are consumed in the protocol, I is the quantum conditional mutual information, and H is the quantum conditional entropy. (If E is negative, then shared entanglement is generated rather than consumed.) The optimal qubit rate provides the first known operational interpretation of quantum conditional mutual information. Their protocol also yields a fully operational proof of strong subadditivity (i.e., nonnegativity of $I(R; C|B)$), because the optimal qubit communication cost of state redistribution cannot be negative.

Making classical honest-verifier zero-knowledge protocols secure against quantum attacks. Zhang, with Hallgren, Kolla, and Sen, showed that any classical Statistical Zero-Knowledge protocol can be modified such that the resulting protocol is still secure, yet is provably secure against any quantum attack [10]. In a Zero-Knowledge proof system, the Prover convinces the Verifier that a statement is true without revealing any further information to the Verifier beyond the validity of the statement. Zero-Knowledge is a well-studied area in theoretical computer science and cryptography, and Zhang’s work establishes a particular setting where classical cryptographic protocols can retain their power even in the post-quantum world.

Sampling of min-entropy relative to quantum knowledge. König, with Renner, showed that quantum conditional min-entropy is preserved under random sampling of subsystems [11]. This generalizes a result previously shown in a purely classical context by Vadhan. It implies that the sample-then-hash approach for generating keys in the bounded storage model is secure even in the presence of a quantum adversary. The proof relies on a novel decomposition of conditional density operators which might have further applications in single-shot quantum information theory.

Cryptography from noisy photonic storage. Wehner, with Schaffner and Terhal, showed that cryptographic tasks can be implemented securely based on the assumption that it is difficult to store quantum states without errors [12]. They considered an explicit noise model inspired by present-day technology, allowing the adversary to extract a noisy record of each incoming qubit, and derived explicit security tradeoffs between the amount of noise that occurs when storing quantum states and the amount of noise the honest parties experience when sending qubits over a channel.

They concluded that security can be achieved if the channel noise rate does not exceed 11% and is strictly smaller than the storage noise rate. They devised protocols that realize secure oblivious transfer and also secure identification (where a party identifies herself without giving away her password).

Quantum entanglement and quantum information theory

Quantum graphical models and belief propagation. Poulin, with Leifer, generalized the concept of graphical models to the quantum setting [13]. Classical graphical models are used to describe a wide variety of inference problems and have applications in numerous scientific fields. Poulin and Leifer proposed a generalized belief propagation algorithm suited for quantum graphical models and characterized its domain of applicability. They also applied the algorithm to the problem of decoding quantum error-correcting codes and to computing correlations of many-body quantum systems.

On exchangeable continuous variable systems. König, with Wolf, studied permutation-invariant Gaussian states and their partial traces, i.e., exchangeable Gaussian states [14]. They gave a complete characterization of the corresponding covariance matrices and derived bounds on the approximation of Gaussian exchangeable states by convex combinations of product states. This result extends de Finetti-type arguments to an important class of states of infinite-dimensional systems.

Entangling power of an expanding universe. Ver Steeg, with Menicucci, studied the entanglement that can be established between two particle detectors that are both moving inertially in de Sitter space [15]. Their calculations revealed that the quantum correlations of the detectors in de Sitter space are distinct from those in hot flat space. In particular, entanglement between the observers in the inflationary universe is suppressed when they are separated by more than a Hubble distance.

Simulating quantum correlations with finite communication. Toner, with Regev, showed that with only two bits of classical communication, two parties can simulate any correlations that could be produced by performing two-outcome local measurements on a bipartite d -dimensional quantum state [16]. All previous protocols for exact classical simulation of these quantum correlations required the communication to grow to infinity with the dimension d . The analysis of the Toner-Regev protocol is based on a power series method (resembling Krivine's bound on Grothendieck's constant) and on the computation of volumes of spherical tetrahedra.

Finite de Finetti theorem for conditional probability distributions describing physical theories. Toner, with Christandl, characterized symmetric states in probabilistic theories, showing that their marginals can be approximated by convex combinations of independent and identical conditional probability distributions [17]. They worked in a general framework where the

state of a physical system is defined by its behavior under measurement, and the relation between different systems is constrained by a no-signaling principle. This result has applications to the foundations of physics, quantum cryptography, and the study of classical channels. In particular, it applies to correlations obtained from quantum states even when there is no bound on the local dimension, a case where known quantum de Finetti theorems are not applicable.

The quantum moment problem and bounds on entangled multi-prover games. Wehner, with Doherty, Liang, and Toner, studied the quantum moment problem: Given a conditional probability distribution, does there exist a quantum state and a collection of measurement operators compatible with the distribution and with a specified set of polynomial constraints? Using recent results from algebraic geometry, they showed that if an instance of the quantum moment problem is unsatisfiable, then there exists a certificate of a particular form that proves unsatisfiability [18]. They applied this result to one-round multi-prover games with entangled provers, showing that a hierarchy of semidefinite programs converges to the entangled value of the game (assuming that the Hilbert space shared by the provers is finite dimensional).

Quantum error correction, fault tolerance, and control

Fault-tolerant quantum computation against biased noise. Preskill, with Aliferis, formulated a scheme for fault-tolerant quantum computation that works effectively against highly biased noise, where phase errors in the computational basis are much more likely than bit-flip errors [19]. In their scheme, the fundamental operations performed by the quantum computer are single-qubit preparations, single-qubit measurements, and conditional-phase (CPHASE) gates, where the noise in the CPHASE gates is biased; they showed that the accuracy threshold for quantum computation can be improved by exploiting the noise asymmetry. For example, if dephasing dominates bit-flip noise in the CPHASE gates by four orders of magnitude, they found a rigorous lower bound on the accuracy threshold higher by nearly a factor of five than for the case of unbiased noise.

Iterative decoding of sparse quantum codes. Poulin, with Chung, studied the problem of decoding a sparse quantum error-correcting code with an iterative algorithm [20], using techniques similar to those that yield state-of-the-art results for decoding classical codes. Unfortunately, the degeneracy of sparse quantum codes badly degrades the performance of the decoding algorithm. Poulin and Chung proposed some methods for overcoming this limitation, which, according to numerical simulations, greatly improve the performance.

Quantum serial turbo-codes. Poulin, with Tillich and Ollivier, presented a theory of quantum turbo-codes and studied their performance numerically for the case of a depolarizing channel [21, 22]. They demonstrated that, in contrast to the classical case, all recursive encoders lead to catastrophic error propagation. This important new feature of the theory leads to new code constructions and methods of analysis. The numerical results indicate that quantum turbo-codes

are the best quantum error-correcting codes found to date, in terms of error threshold for a given transmission rate.

Topological cluster state quantum computing in three dimensions. Goyal, with Fowler, used the stabilizer formalism to reformulated and reanalyze protocols for fault-tolerant “one-way” quantum computation using three-dimensional cluster states [23]. Their new analysis confirmed the principal conclusions found earlier by Goyal with other collaborators, specifically a threshold error rate approaching 1% and arbitrarily long-range logical gates with low overhead. Furthermore, this analysis applies to a broader range of quantum computing technologies.

Discrete approximation of quantum stochastic models. Bouten, with van Handel, developed a general technique for proving convergence to the solution of a quantum stochastic differential equation for a quantum system that interacts repeatedly with a noise source [24]. This method has wide applicability, as they illustrated by analyzing a variety of examples. In contrast to previous work, their main theorem is not restricted to a specific noise model and does not require boundedness of the limit coefficients.

Approximation and limit theorems for quantum stochastic models with unbounded coefficients. Bouten, with van Handel and Silberfarb, proved a limit theorem for quantum stochastic differential equations with unbounded coefficients, extending the Trotter-Kato theorem for contraction semigroups [25]. Using this theorem, they obtained general results on the convergence of approximations and singular perturbations, which they illustrated by applying to several examples of physical interest.

Optimality of feedback control strategies for qubit purification. Bouten, with Wiseman, proved the optimality of control strategies for rapid purification of qubits that had been proposed earlier by Jacobs and by Wiseman and Ralph [26]. Their proof uses simple concepts from optimal control theory, including Bellman equations and verification theorems.

Adiabatic elimination in quantum stochastic models. Bouten, with Silberfarb, studied a model of a physical system coupled to a bosonic reservoir via a quantum stochastic differential equation [27]. They showed that in the limit of infinite coupling strength the solution to the quantum stochastic differential equation converges strongly to the solution of a limiting quantum stochastic differential equation. In the limiting dynamics the excited states are removed and the ground states couple directly to the reservoir.

Experiment and implementation

Quasiparticle poisoning and Josephson current fluctuations induced by Kondo impurities. Kitaev, with Faoro and Ioffe, introduced a toy model to study the physical properties of a spin impurity coupled to the electrons in a superconducting island [28]. They showed that when the coupling of the spin is of the order of the superconducting gap, two almost degenerate subgap

states are formed. By computing the Berry phase that is associated with the superconducting phase rotations in this model, they proved that the switching between these subgap states has the same effect as quasiparticle poisoning of the island. They also showed that an impurity coupled to both the island and the lead generates Josephson current fluctuations.

Interferometry of non-Abelian anyons. Bonderson, with Shtengel and Slingerland, used topological field theory methods to develop a general theory of anyon interferometers [29]. They applied their new techniques to describe interferometry measurements in a double point-contact interferometer that can be realized in fractional quantum Hall systems. The measurements they proposed might soon be performed, an important step toward the realization of a topological quantum computer that is intrinsically resistant to decoherence.

Universal periods in quantum Hall droplets. Refael, with Fisher and Fiete, analyzed universal-period conductance oscillations in finite size quantum Hall interferometers, which had recently been cited as the first direct experimental evidence for fractional indistinguishable particle (anyon) statistics. They pointed out that when one quantum Hall state is engulfed by another, periodic conductance oscillations can occur that have nothing to do with quantum interference, but instead arise from size modulations of the quantum Hall droplets [30]. Though it threw cold water on a potential experimental breakthrough, this work provides valuable guidance for future experiments that will probe anyon statistics and pursue new schemes for topological quantum computing.

Noncommuting flux sectors in a tabletop experiment. Kitaev, with Moore and Walker, showed how to use superconductors and Josephson junctions to explore the groundstates of the free electromagnetic field in a 3-manifold whose cohomology contains torsion [31]. Moore and others had shown previously that the electromagnetic field on such a 3-manifold has a degenerate ground state described by a representation of the Heisenberg group. These manifolds cannot be embedded in Euclidean three-dimensional space, but they can be immersed with self-overlaps, and Kitaev *et al.* found that the immersion can be mimicked using superconducting devices.

Quantum state engineering and precision metrology using state-insensitive light traps. Kimble, with Ye and Katori, reviewed recent experiments that use the tools of cavity quantum electrodynamics for coherent preparation of quantum states of laser-cooled optically trapped neutral atoms [32]. In these experiments, two electronic states of an atom experience the same trapping potential, permitting coherent control of electronic transitions independent of the atomic center-of-mass motion. These methods have applications to frequency metrology, optical atomic clocks, quantum networks, and quantum computation.

Heralded entanglement between atomic ensembles. The Kimble group established heralded entanglement between collective excitations in two atomic ensembles, using a probabilistic protocol [33]. To verify the entanglement, they then converted each collective excitation to the field of a single photon, and measured the concurrence of the resulting optical state. From this

measurement, they inferred a lower bound 0.9 ± 0.3 on the concurrence of the collective atomic state. They also studied the decay of entanglement as a function of storage time, and interpreted this decay using a model of the local dynamics.

Mapping photonic entanglement into and out of a quantum memory. The Kimble group [34] established entanglement between two atomic ensembles by coherently mapping an entangled state of light. In contrast to previous work, the entanglement was established in a deterministic protocol rather than a probabilistic one. After a programmable delay, they mapped the stored entanglement back into photonic modes with overall efficiency 17%. With improved single-photon sources, this protocol should be able to achieve “on demand” entanglement of atomic ensembles, a powerful resource for quantum networking.

Connecting quantum information with the rest of physics

Black hole information retention time. Preskill, with Hayden, studied information retrieval from evaporating black holes, using tools and results from quantum coding theory [35]. First they noted that, if a black hole’s internal degrees of freedom are nearly maximally entangled with the previously emitted Hawking radiation (as would be expected for a black hole that has already radiated away more than half of its initial entropy), and if the internal dynamics of a black hole is rapidly mixing, then k qubits of quantum information dumped into the black hole will be revealed after just a few more than k qubits are emitted in the Hawking radiation. Next they applied the recently developed theory of approximate unitary two-designs to estimate a black hole’s thermalization time, concluding that rapid thermalization is plausible. Remarkably, the resulting estimate of the black hole information retention time is just barely long enough to be compatible with the black hole complementarity principle.

Belief propagation algorithms for quantum many-body physics. Poulin and Bilgin have numerically investigated the performance of quantum belief propagation for the computation of correlation functions of finite-temperature quantum many-body systems on loopy graphs [36]. Previously proposed efficient algorithms for this task (such as the density-matrix renormalization group method) work only for tree graphs. But in the classical setting, belief propagation is reliable on graphs with loops provided all the loops are large, and thus has been applied successfully to important problems in coding theory and statistical physics. The results of Poulin and Bilgin indicate that quantum belief propagation also works effectively for graphs that do not contain small loops.

Topological multicritical point in the toric code and 3D gauge-Higgs Models. Exploiting insights derived from the toric quantum error-correcting code, Kitaev, with Tumpitsyn, Prokof’ev, and Stamp, has discovered a new type of multicritical point that arises from competition between the Higgs and confinement transitions in a Z_2 gauge system [37]. By Monte-Carlo

simulations, they mapped out the phase diagram of the 3d gauge-Higgs model, finding second-order Higgs and confinement transition lines that merge into a first-order line. A similar phase diagram is predicted for the two-dimensional quantum toric code model with external fields that drive the formation of a charge condensate (resulting in vortex confinement) or a vortex condensate (resulting in charge confinement). These findings pose the challenge of finding an effective field theory, presumably involving two noncommuting order parameters, for the multicritical point.

Quantum reference frame and deformed symmetries. Poulin, with Girelli, suggested a simple physical mechanism supporting the proposal that quantum fluctuations of the spacetime metric can be described by an effective theory with a deformed Lorentz symmetry [38, 39]. Their argument is based on the analogous phenomenon that occurs when a quantum gyroscope is used to measure the spin of a particle. The analysis also places bounds on the detectability of such a symmetry deformation.

Non-Abelian statistics in the interference noise of the Moore-Read quantum Hall state. Ardonne, with Kim, proposed noise oscillation measurements in a double point contact device that are feasible with current technology, which could confirm the non-abelian nature of fractional quantum Hall state with filling factor $5/2$ [40]. By calculating the voltage and temperature dependence of the current and noise oscillations, they predicted that the non-abelian state can support two qualitatively different types of frequency dependence of the noise. The proposed experimental test can also be adapted to other non-abelian fractional quantum Hall states.

Collective states of interacting Fibonacci anyons. Motivated by a desire to determine how anyon interactions might impact the operation of a topological quantum computer, Ardonne, with Trebst *et al.*, has shown that chains of interacting Fibonacci anyons can support a wide variety of collective ground states [41]. They found that energetic competition between two-anyon and three-anyon interactions can lead to a rich phase diagram harboring multiple critical and gapped phases. They confirmed numerically that the critical phases (and their higher-symmetry endpoints) can be described in terms of two-dimensional conformal field theories. A topological symmetry protects the critical phases and constrains the properties of the gapped phases.

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